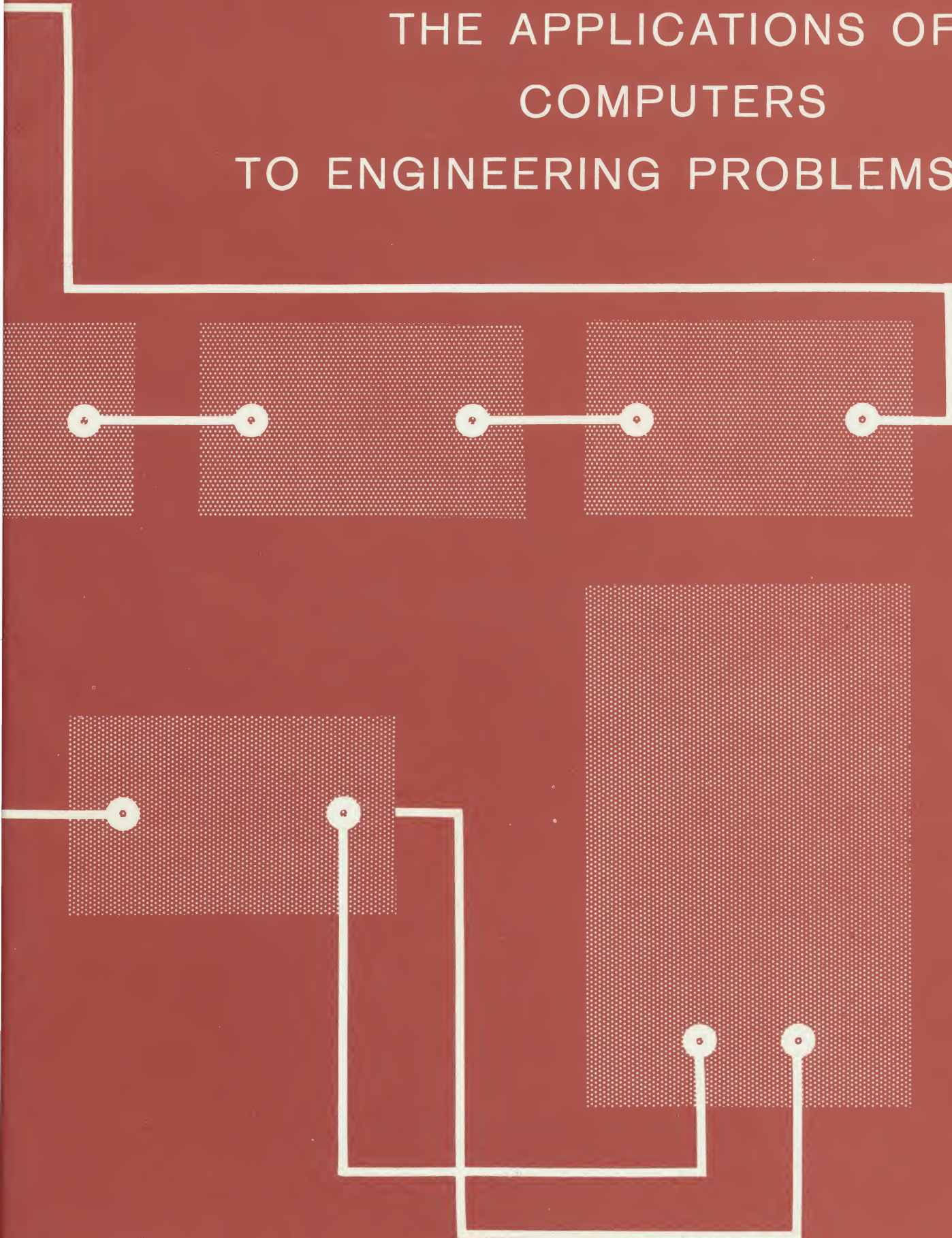


'PERSONAL ANALOG COMPUTER.'

five lectures introducing

THE APPLICATIONS OF COMPUTERS TO ENGINEERING PROBLEMS



The Applications of Computers to Engineering Problems

This is an abbreviated course on the analysis and design of engineering systems using the Personal Analog Computer. This paper is divided into five lectures, each to be given with the Personal Analog Computer in hand. It is not directed towards any special discipline, but rather covers those aspects all disciplines have in common, namely the transmission and utilization of power and information around loops and in control. The student studies simultaneously the great generalities of nature, and the specific measurements from his computer model, hopefully achieving a balanced and impressive picture of the real world.

ABSTRACT

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Introduction

This series of five lectures is organized to be used with the Personal Analog Computer to introduce modern engineering systems analysis. The concepts and spirit have been borrowed from a graduate course in Mechanical Engineering designed to update practicing engineers.* Although these lectures can only touch upon a limited number of problems, they should give the student an excellent picture of the kind of revolution the computer has brought about in modern systems engineering.

The computer has made it possible for an engineer to model a complete system prior to actually building, and to investigate its features. Such electronic models are now commonplace. Prior to an expensive capital investment in a plant, or water system, or other such fabrication, an engineer can make a computer model and study exhaustively the various features of the system. The importance of such simulations to industry is obvious and has been dwelt upon even in the popular literature.

A much less publicized benefit of computer modeling has been the broad insight such models have provided in the general field of system design. It became obvious that people in widely different fields were studying basically the same systems. Thus the idea of interdisciplinary studies stressing the common features of mechanical, electrical, hydraulic, and other systems sprung up. The modern engineer must be able to deal in a diversity of fields. Learning these common system problems once rather than repeating their solutions in different courses with different names and components seems to be much more efficient.

A second discovery which has been suggested in a few very advanced schools is that the old fashioned method of system design, which in practice involved a mathematical and analytical solution of a system, if possible, or the empirical copying or modifying of systems already built, is now giving way to the computer approach whereby the designer goes directly from the requirement to the computer block diagram or program, and thence to the system simulation or design, using modern operational symbols and essentially solving no mathematical problems personally. Some people suggest that com-

* Henry M. Paynter has developed and used the system classification and notation suggested here in a highly successful one-semester course for senior and graduate students at the Massachusetts Institute of Technology. To appreciate the real power of Professor Paynter's approach and the breadth of this subject, the reader is referred to his Class Notes for Course 2.751 *Analysis and Design of Engineering Systems*, and his forthcoming book, *Ergs and Bits: The Flow of Energy and Signals in Engineering Systems* to be published by the McGraw-Hill Book Company.

puter symbology may ultimately replace classical mathematics in the description and solution of problems.

In summary, the use of the Personal Analog Computer in a study such as this one would have many benefits:

It would stress the interdisciplinary aspects of systems, emphasizing those features common to all physical systems, and deemphasizing the special names, symbols and units associated with one particular subject.

It would provide an approach to the synthesis and analysis of physical systems which is basically intuitive involving cause and effect concepts which are fundamental to the human understanding, and requiring a minimum of specialized mathematical or physical training.

It gives the student an actual tangible model to work with. It is simultaneously a model of the mathematical expressions, and the physical system. It is a model which breaks down into the basic elements whose behavior is almost intuitive, and can be assembled into complex systems that are totally unanalytic in the mathematical sense.

Finally it permits the student to go much further into the subject of systems on an individual and experimental basis. He doesn't have to learn on the basis of memorizing and authority, which is both inefficient and unreliable, but he can find out for himself with personal and direct experiment using his own equipment.

Lecture #1 deals in the anatomy of systems. It illustrates how to break physical systems down into variables, operational elements, and interconnections. The power variables "effort" and "flow" are suggested as fundamental to all physical systems. Their dynamic relationship in a system element determines whether that element is resistive in nature or capacitive or inertial. Discussion is limited to these three elements although they are not required to be linear. The manner of interconnection is then taken up, and there are shown to be two types of junctions: one where flow is common to the connected elements, and total effort sums to zero; and the other where effort is common to adjoining elements and flows sum to zero.

Lecture #2 studies the familiar mass, spring, dashpot problem in two different interconnections, one with effort summing, and one with flow summing.

Lecture #3 deals with a system which is more complicated and has two junctions.

Lectures #4 and #5 investigate a non-linear problem completely. Although the problem is relatively simple, it has a surprising number of ramifications, and it cannot be solved by pure analytical methods.

The Anatomy of a System

When engineers study physical systems they often find that the whole is much more complicated than the sum of its parts. They have accurate specifications of the relays, motors, and pipes, etc., which the system will contain, and yet remarkable and unpredictable things occur when these components are interconnected. This complexity is the basic subject matter of system engineering.

A classical example of systems analysis was the mathematical modeling of the planetary systems by Newton. The behavior of the planets had baffled men for ten centuries and had produced only bizarre explanations such as spheres mounted on spheres. Newton made use of component specifications such as the way which masses accelerated under force, and the relationship between the gravitational forces and distances between masses, and formulated a complete description of the behavior of the planets.

Today lesser men tackle far more ambitious problems in systems analysis which include such systems as the human body, where the components are not even defined, but are hypothesized, and the measurements are statistical. In spite of our modern sophistication, however, the general public tends to underrate or ignore completely the subtle effects of interrelations between components.

To illustrate a very simple example of system subtlety we introduce one of the PAC units, the Adder, and show how it works. It adds all of the inputs to it, changes the sign of this sum, and adds another input generated by the knob on the front. With no inputs, it is possible to set the output to any value, say 8. Now what will happen when the output is connected to the input. Not many people can tell in advance, even though they thoroughly understand the operation of this component prior to this connection.

A FUNDAMENTAL APPROACH

In either the analysis or synthesis of a system, the first step is to break the system down into known parts. We must choose the variables which we are concerned with, we must summarize this information by a block diagram showing the variables, the components and their interconnections. It is possible to reduce most physical systems to relatively few basic elements, variables, and connections, particularly if the variables are properly chosen. We will not illustrate all of these possibilities, by any means but will confine the description to relatively simple elements.

THE POWER VARIABLES

In physical systems the quantity we propose to work with is power. Power is the product of two variables, which are those we usually measure with our instruments. In general terms we are calling them the effort and the flow. Now, in different physical systems, these have different names. In electrical systems, the effort variable is voltage and the flow is current. In mechanical systems the effort variable is force and the flow is velocity. The Figure 1.1 shows the effort and flow variable names in different types of systems. Their product is always power.

General	Mechanical	Electrical	Fluid	Rotational	Thermal
effort	force	voltage	head	torque	temperature
flow	velocity	current	flow	angular velocity	entropy flow
resistance	friction	resistance	resistance	friction	insulation
capacitance	spring	capacitance	tank storage	angular spring	specific heat
inertance	mass	inductance	momentum	angular momentum	

Figure 1.1
Variables and Elements in
Different Systems

THE ELEMENTS

An element in a system is defined by the relationship between flow and effort associated with that element. We will define three types of elements: They are most obvious in electronic systems, where we can immediately say that an element is resistive, capacitive or inductive depending on the relationship between current and voltage. Similar elements exist in other systems.

The general resistive element is one where the effort and the flow are related by a constant. It may not necessarily be linear. Thus in a fluid resistance is nonlinear.

When the effort variable in an element is proportional to the integral of flow, we can generally define it as a capacitive element. A condenser integrates current. Water stores heat. This capacitive relationship does not have to be linear, it may even have dead zones, as does water at freezing.

When the flow is proportional to the integral of effort, we call the element an inertance. This is the case of a mass whose velocity will be the integral of the force. Figure 1.1 shows how these three types of elements appear throughout physical systems under different labels.

THE JUNCTIONS

When we have reduced a complex system to a number of effort and flow variables, and a number of elements of resistance, capacitance and inertance, it is still necessary to show their interconnection. We recognize immediately two basic types of connections between two or

more elements. They are most obvious from the viewpoint of electronics where they are called serial and parallel connections. In a serial electrical connection, the current is common to all elements, and the voltages across the system elements sum to zero. This is called Kirchhoff's law. We call this an effort summing junction. There is another connection where the elements are in parallel, and have common effort, but the currents or flows sum to zero. This is a flow summing junction.

The junction concept is more subtle in other systems, but no less important to identify. With a mechanical system such as a mass and spring, the velocity of spring extension is the same as the velocity of the mass, but the sum of the forces are zero. It is a force or effort summing junction.

THE BLOCK DIAGRAM

A block diagram is a clear way to represent a system. It shows each of the elements and their interconnections. The blocks can be labeled in terms of how they operate on the variable mathematically, and also what physical part of the system they represent. Variables can be labeled on a block diagram, and are indicated by lines interconnecting the blocks.

Figure 2.1 in the next lecture illustrates a typical system. It is reduced to block diagram form and the variables of effort and flow are paired to indicate power flow. It is evident that this block diagram form will lead directly to a computer program, or model.

The Personal Analog Computer is designed for direct substitution into the block diagram representation. The Coefficient Multiplier represents a dissipative element. It can accept either a flow or effort variable, and convert it to the other with the proper coefficient setting or series of settings. The Integrator unit represents either a capacitance or an inductance depending upon whether flow or effort variables are feeding in. The Adder unit represents either a flow summing junction or an effort summing junction, again depending on what variables it is adding up.

The Personal Analog Computer units are unilateral. Signals go through one way. Each unit has inputs and outputs. One can think in terms of cause and effect, where the input is the cause, the output is the effect, and the element a cause-effect relationship. If the system is divided into flow and effort variables, we can usually intuitively associate a similar cause and effect relationship to its elements. A spring, for example, receives a velocity input which it integrates to yield a force as a result. You can't put force on a spring, any more that you can impress voltage on a condenser. A mass has the dual

relationship of accepting force and accumulating velocity as an effect of force.

Fig. 1.2 illustrates how PAC units simulate the elements and junctions in a system.

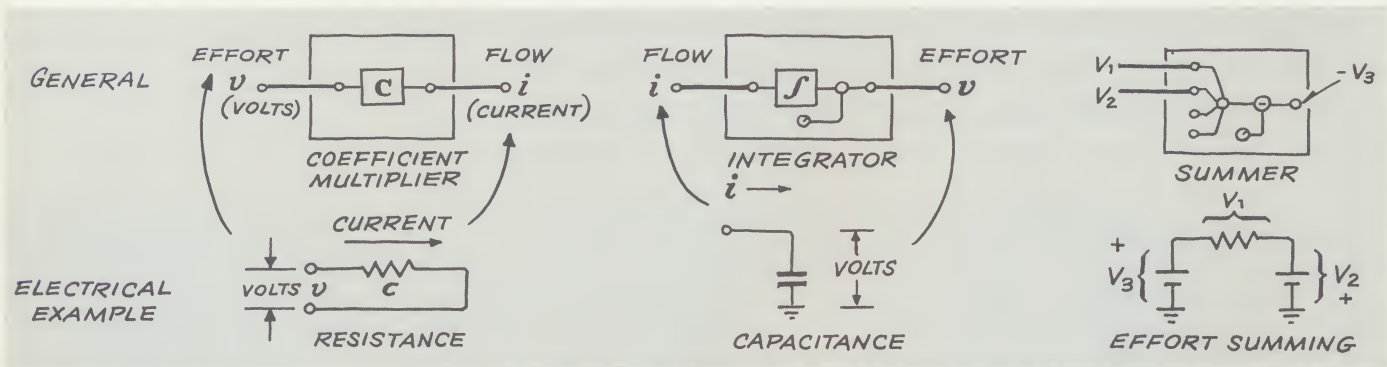


Figure 1.2
How PAC Units Represent
Elements and Junctions in a
System

SUMMARY

In this lecture we have started by accepting the fact that systems tend to be more complex than the mere consideration of their components would lead us to believe. We have therefore recognized the virtue of making a model of a complicated system for testing and modifying our theories, prior to the actual fabrication of the system. We have also recognized the value of modeling existing systems, because of the ease of manipulating computer models.

In order to analyze a system in this way we must first isolate the variables in the system. We can choose the variables to be either an effort or flow variable.

When the effort and flow variables are identified, there will be three types of elements relating each effort to each flow. These elements are called in general terms, resistive, capacitive, or inertial.

The elements will be interconnected at power junctions through which pairs of flow and effort pass. There will be two types of junctions, flow summing junctions, where all elements have common effort, and their flows add up, and effort summing junctions where all elements share the same flow, but their efforts add up.

How System Elements and Variables Are Modeled by Equations & Computers

REVIEW:

In the previous lecture, we agreed that a good basic approach to systems analysis was to choose variables, and then operations which related these variables, which we identified with elements in the system. We decided that the two variables of effort and flow were always present in systems where power was exchanged, and that such systems were inclusive enough to be worth studying. We note that sometimes we are actually concerned with only one variable representing information and can neglect the power problem. This is a special case, however, included in the systems we study now.

The variables of flow and effort are relatively easy to perceive in most systems, and readily measured. Power, their product, is more elusive. When we see a shaft revolving, we see flow. We usually cannot tell if it is exerting effort or therefore transmitting power. We can see water flowing, and measure current flow and voltage (effort) in electrical systems. We can feel temperature and appreciate that this will exert an effort on heat flow, but again the power transmission is subtle because the flow is hard to sense. In any case, these variables which are of concern in real systems can be expressed in terms of effort and flow.

These two variables of effort and flow have led us to the three types of elements the first of which was pure resistance, or what is sometimes called a dissipative element. Dissipative elements are by no means undesirable in all cases in spite of the connotation. The term dissipation means that power leaves the system which is often what you want. For example in a heating element, you want your effort (volts) and flow (current) to go directly into heat. The undesirable case, such as friction in the automobile is again a translation of the effort and flow into undesirable heat, instead of power in terms of automobile velocity and force. The PAC Coefficient Multiplier is a model of this relationship.

The second element is the capacitance. It is an element where a flow is converted into an effort according to the relationship of:

$$e = \frac{1}{C} \int f dt$$

A capacitance is something that you apply flow to, but not effort.

You cannot get an effort out of capacitance instantaneously unless you have infinite flow. A good example of this is a spring. You cannot get an effort from the end of a spring until you distort it. This takes time unless you move with infinite velocity (velocity is flow). If you apply a fixed velocity to the end of spring, the force increases linearly with time. Another example is a water barrel, where the pressure of water at the bottom is the effort. If a flow of water is steady into the barrel, the effort increases with time. The PAC Integrator is a model of this relationship.

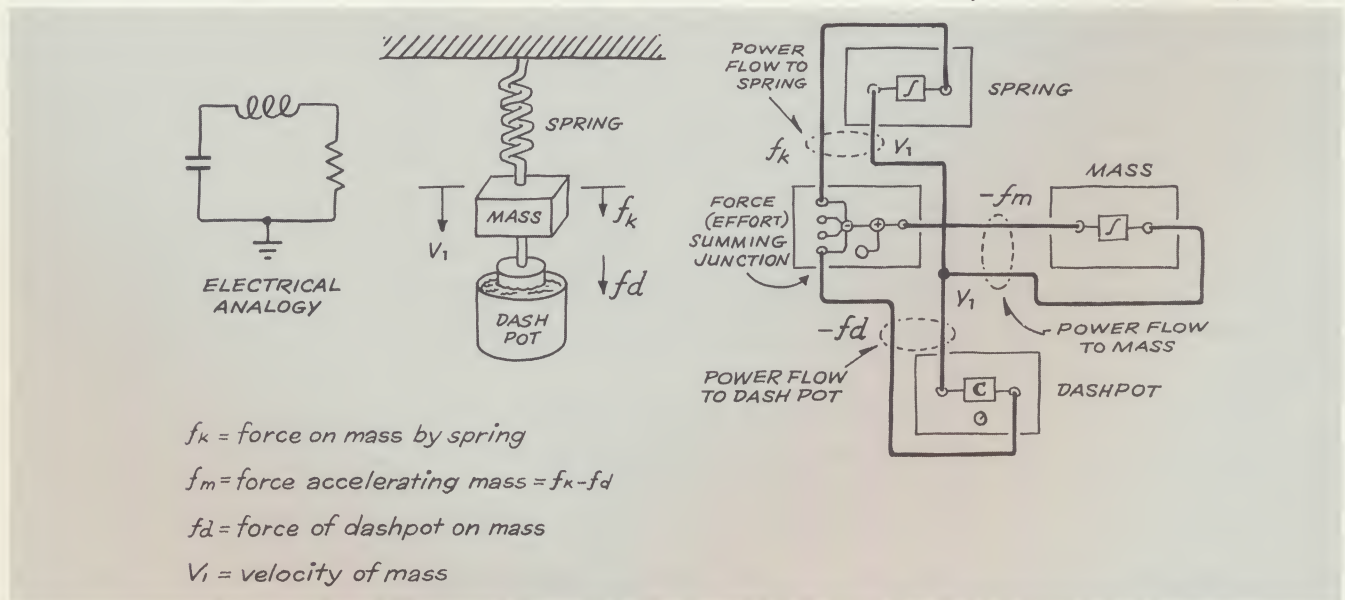
A final relationship is an inertance, this word being used to represent a resistance to flow. It is the inverse of Capacitance, and expresses the element that integrates effort to yield flow. An example is a mass. It integrates force to generate velocity. Another is inductance, which integrates voltage to yield current. One always has effort feeding into Inertances and resulting in flow.

Notice the three input-output plots associated with these elements. We have not insisted that the relationships be linear, although the PAC units are linear. We do require that they be single valued, however, and have no hysteresis. Now let us investigate a classical mechanical system in which the elements appear in relatively pure and isolated forms.

A SYSTEM MODEL

Figure 2.1 is a mechanical system consisting of three mechanical elements. The mass, the spring, and the dashpot. We want to model this system using the PAC units. This should be relatively straightforward since we have already modeled these individual components with PAC units. However, as usual, the system is more complicated.

Figure 2.1
Spring Mass Dashpot System



It is not obvious how the PAC units should be interconnected. Should their interconnections bear some one to one correspondence with the physical connections of the mass and spring and dashpot? The answer usually is no.

The proper beginning is to define the variables of effort and flow which in this mechanical example are force and velocity. There are two forces, one exerted by the spring on the mass, and one exerted by the mass on the dashpot. There is evidently only one velocity, that of the mass, which is common to the dashpot, and spring extension.

This illustrates a kind of junction where velocity or flow is common and forces or efforts add. It is analogous to a series connection in electric circuits where the voltages add up to zero, and the current is common to all elements. We therefore use an Adder Unit to sum forces.

In figure 2.1b, the Adder Unit is shown adding the two forces to yield the force on the mass. When we Integrate f_m as shown we get the common velocity v which integrated again gives us the spring force, and when this velocity is multiplied by a constant it gives us the dashpot force. These are the inputs to the Adder.

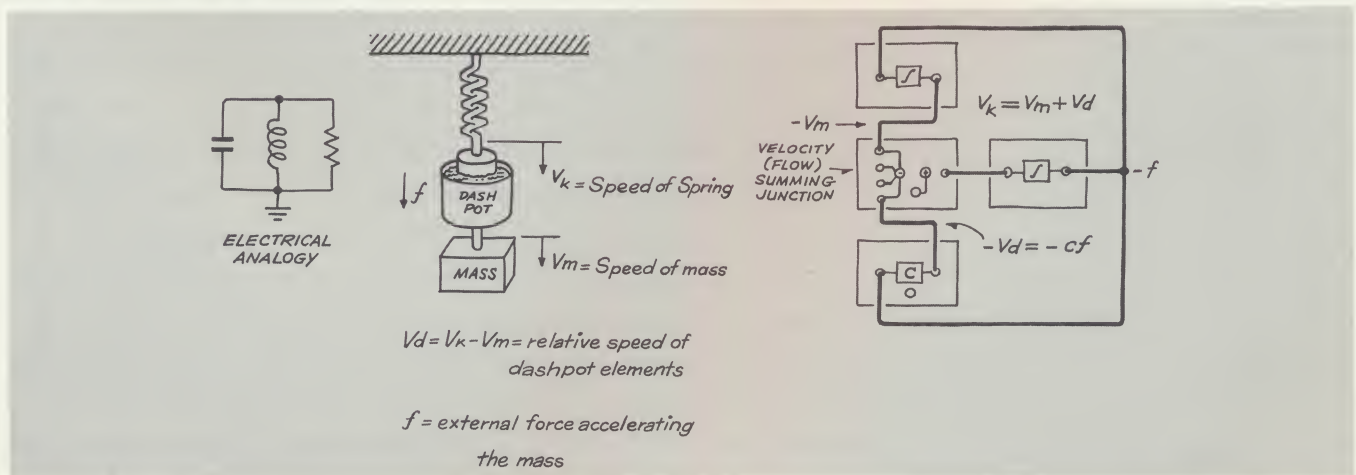
A "FLOW SUMMING" JUNCTION

We have seen how a system is modeled which has one junction of the "effort summing" type, and a resistance capacitance and inductance element. For completeness we will illustrate the other type of junction with the same elements.

Figure 2.2a is a "flow summing" type of junction. A single force is common to all elements. This is true because the mass no longer divides the elements, and a mass is capable of modifying a force.* If there were another mass attached between the spring and the dashpot, there would be neither a common velocity nor a common

* This is why some people buy big cars.

Figure 2.2
Spring Dashpot Mass System

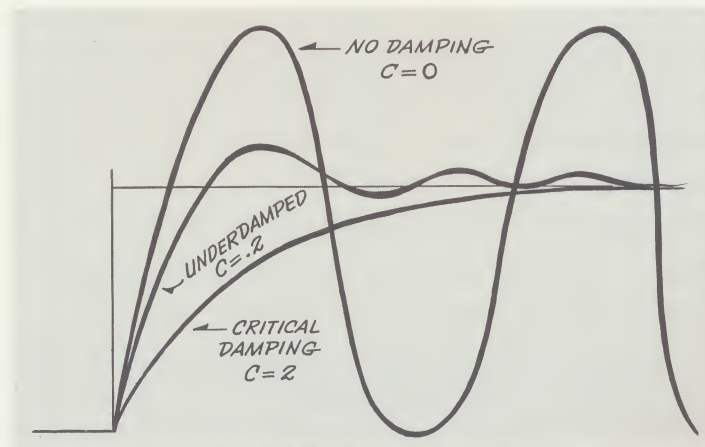


force. Is this some other kind of junction? No. It is two junctions. It would require two Adders for its simulation. The next lecture deals with a two junction System.

Returning to Figure 2.2b the Adder will now add velocities. We would like the Adder output to be V_k since this is integrated to give us the common force. Then the inputs must be V_m and $V_k - V_m$. These are obtained by integration of force and multiplication of force by a constant, representing the behavior of the mass and the dashpot respectively.

These illustrations with PAC units give completely linear solutions showing in Fig. 2.3. They can therefore be solved by pure mathematical considerations, and if we write the differential equations governing the variables we can arrive at analytical solutions. In general, however, the relationships will not be linear and the integrators will be succeeded by some sort of nonlinear function generator representing the actual relationship between variables. With the PAC units these relationships can sometimes be incremental approximated as will be illustrated in lectures #4 and #5.

Figure 2.3
Solutions to the
Spring-Mass-Dashpot Problem



A Two Junction System

The previous lecture described a single junction system connected up in two ways, common flow and common effort. In general systems will have a number of junctions, and their behavior can become very complicated indeed. Figure 3.1a shows a two junction system consisting of two springs and two masses. For simplicity, only one parameter, that of the bottom mass, is different from 1.

The PAC system in Figure 3.1b has had the variables assigned to it. The first spring and mass are a common flow system since the end of the spring and the mass have a common velocity. This is illustrated in the block diagram by using the Adder to add forces, its output being the sum of forces applied to the mass. The common flow of this junction is applied to the bottom junction, which is "flow summing." The velocity which comes from the top system sums with the velocity of the bottom mass in the second Adder unit, and results in an output which integrated yields the common force or effort in the bottom system. This effort is returned to the top mass.

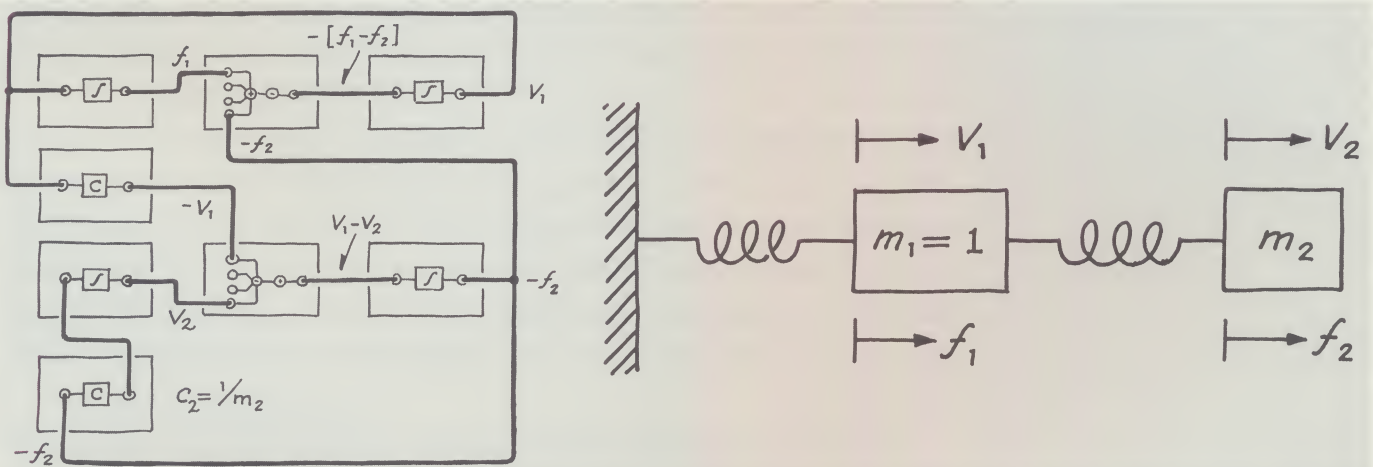


Figure 3.1
A Two Junction System

Notice that we can identify the junction between the top mass and the bottom spring as a connection between the two systems, or a port through which power flows. The flow variable of this power is the velocity which the mass applies to the spring, while the effort variable is the force which the bottom spring returns to the top mass.

It is always confusing to follow signs correctly and great care must be taken to keep sign conventions straight. In nature there always seems to be negative response to various actions, which people summarize loosely by saying that every action has an opposite reaction. In the block diagram any loop must have a negative sign, or trouble will result, and an error in sign has probably been made.

Modeling a Non Linear System

INTRODUCTION

It is the intent of the next two lectures to study exhaustively a single problem with the Personal Analog Computer. This exercise will be particularly instructive in that it will accomplish the following:

1. It will illustrate how, with a computer, one can predict the behavior of a complicated system which cannot be predicted analytically.
2. It will show how a great many interesting things about a system can be revealed through a computer study that are not immediately evident from purely mathematical study.
3. It will show how to attack a problem and to reduce the number of parameters for computer consumption.
4. It will indicate how general solutions can be achieved if a whole class of such problems must be attacked.
5. It will test the judgment and ingenuity of the experimenter in optimizing the accuracy of his computation.
6. It will give the student the opportunity to choose between purchasing special equipment, or doing more work in his computation.
7. It should clarify the relative advantages of analog and digital methods of computing.

All of these things are useful in the solution of every engineering problem. Because of time or equipment limitations, engineers seldom get a chance to appreciate these many advantages of a coordinated system study using a computer.

THE PROBLEM

The problem to be studied is illustrated in Figure 4.1. Two cans have holes in the bottoms and different dimensions. One can is placed above the other, and filled up to a certain level with water. This leaks into the second can and commences to fill it. If the second can is the right size, it will not overflow, but will begin to leak out at a fast enough rate so that it does not quite overflow. The problem is to state the conditions under which the top can will completely fill but not overflow the second can.

A Study Of One Can

Let us start with one can only. Figure 4.2 shows this one can.

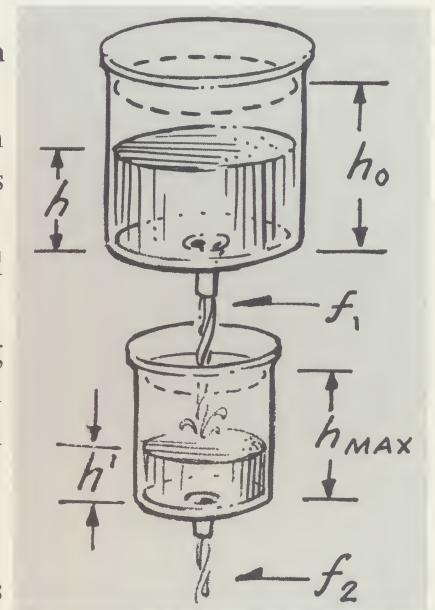


Figure 4.1
Two Can Problem

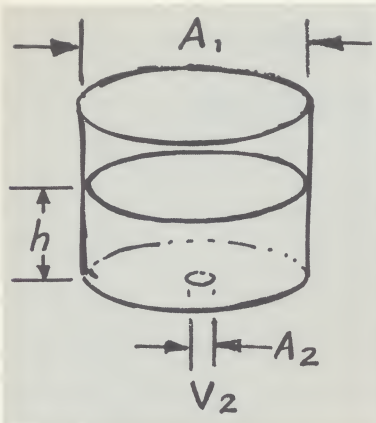


Figure 4.2
A Single Can

It is a cylinder, and we must determine the variables. If we imagine water flowing out of the hole, we can visualize immediately two things changing, the height of the water in the can, and the rate at which it flows out the bottom. If one is an electrical engineer, the immediate reaction is to recognize the analogy between the height of the water, and the voltage on a condenser, and the flow of water, and the current. Certainly the amount of water which flows out of the bottom of the can is proportional to the change in height, just as charge flow affects the voltage. In our more general system's variables the height of head of the water is the effort variable, and the leaking, the flow variable. They are related by an integral relationship indicating a capacitance type of element.

The mathematical equation relating flow and height is derived from a knowledge of physics. It is evident that the volume of water disappearing from the top of the can: $A_1 dh/dt$ must have squirted out of the bottom with a velocity V and volume $A_2 V$. Therefore, by $A_1 dh/dt = A_2 V$.

A second relationship results from the energy considerations. The total energy of a volume of water of mass m on the top where it is almost still, is nearly all potential energy mgh which changes to kinetic energy $\frac{1}{2}mv^2$ when it reaches the hole. Therefore, the velocity at the hole is related to the height by the relationship

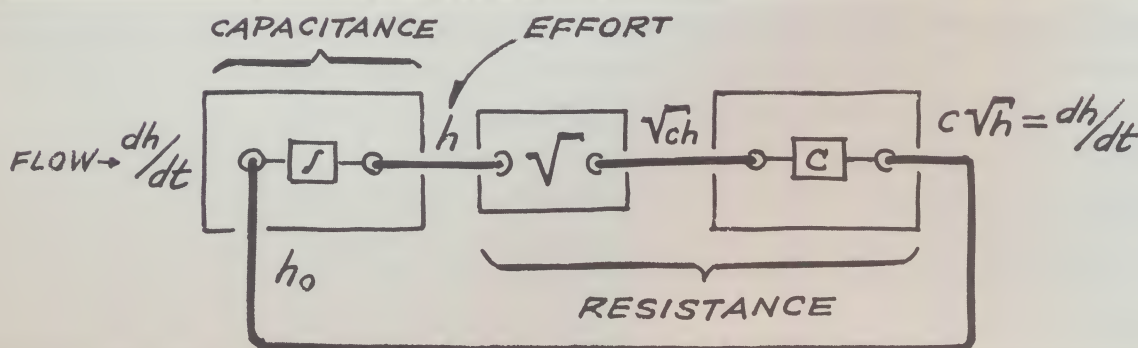
$$\frac{1}{2}mv^2 = mgh \quad V = \sqrt{2gh}$$

These two relationships define the block diagram shown in Figure 4.3. Notice that all of the parameters of the can are tied up in a single constant C . We can classify all cans by C now, and even though they may look different, they have the same behavior. Furthermore, since the C and the Integrator in combination make an Integrator of a different time constant, all cans will have the same fundamental behavior in terms of height vs. time, if the time dimension is adjusted to match C .

Figure 4.3

Block Diagram of a Single Can

The things to determine on the Computer are height of water vs. time and flow vs. time.



COMPUTING THE SQUARE ROOT

Now comes a major problem. There are no square root units in a standard PAC set, and the student must decide whether to purchase one, or to solve for the square root through ingenuity. We will assume that he decides to do the latter, and in Figure 4.4 is shown a square root calculator.

The principle behind this method is that successive integrations of a constant yield a function $x(t)$ which changes linearly with time t and a second function $x(t^2)$ which varies with t^2 . We can run these Integrators until $x(t^2)$ has the desired value (which may be h in the can problem) and feed its square root $x(t)$ (which would be $\sqrt{h}$) into the problem. This computer will require a second PAC set, which is operated separately with only the ground connection common to the first.

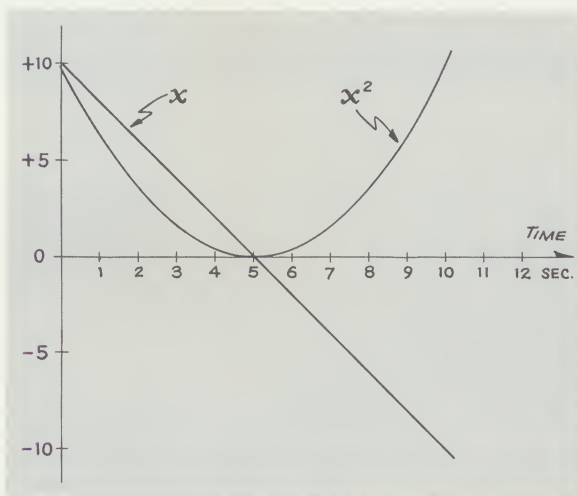


Figure 4.5
Square Root Output Plots

In Figure 4.5 we have plotted how the square root computer reads out with time. We calibrate it by setting the Constant into the first Integrator to $+2$, so that the first Integrator output changes slowly. Set the Initial Condition of both Integrators to $+10$. Set Coefficient to about $+0.4$. Now run the system continuously and observe if x^2 behaves properly, i.e., goes down exactly to 0 and then rises again. If it goes negative, increase C . If it doesn't reach 0, decrease C . If it exactly reaches 0 and reverses, you are assured that the two Integrators have values that are exactly related by a square root relationship.

Full Scale of both functions represent 1, since:

$$x(t) = \frac{x}{10} = +1 - 0.2t$$

and

$$x(t^2) = \frac{x^2}{10} = +1 - 0.4t + 0.04t^2$$

so that

$$x(t) = (x(t^2))^2$$

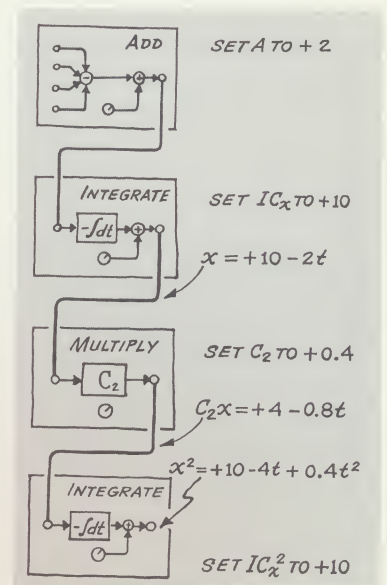


Figure 4.4
Square Root Computer

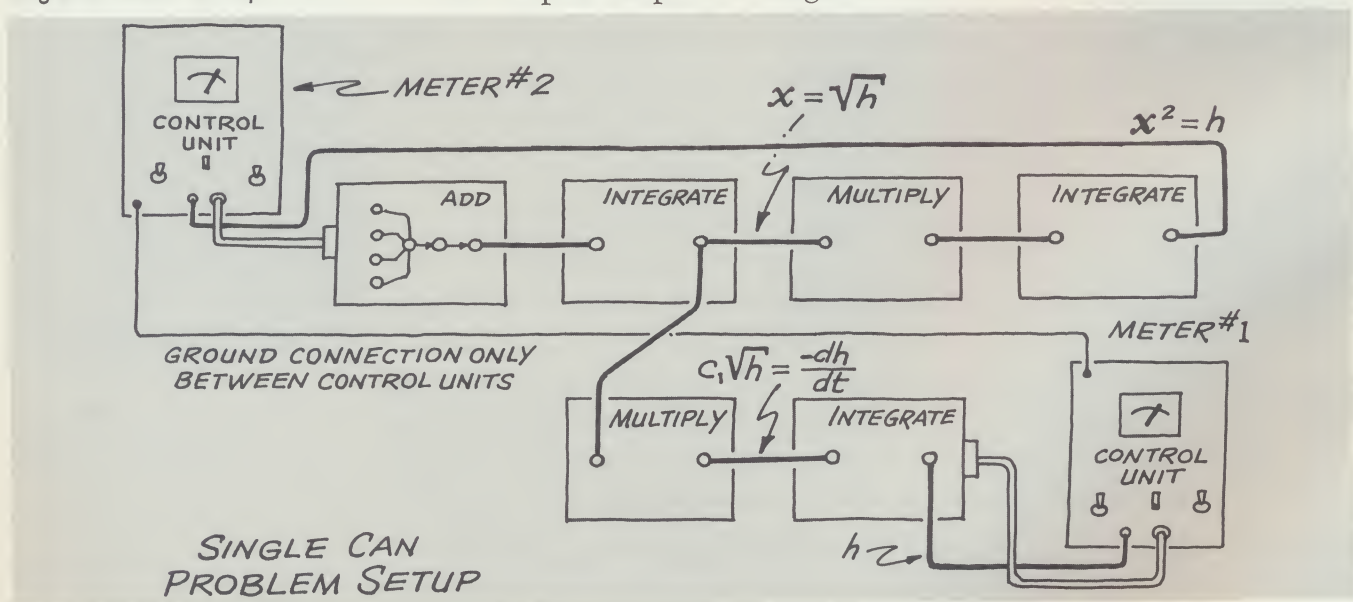
Some Sample Solutions

In Figure 4.6 we have the entire Single Can PAC computer. With Meter #1, read h ; and with Meter #2, read x^2 from the square root computer. We will assume that the constants associated with the can analog add up to 1. Then the height Integrator is fed by $C_1 \sqrt{h} = -dh/dt$ from the square-root computer, and x^2 is made to track the Integrator's output h .

OPERATION

1. Set up the Square-Root computer as described above.
2. Set the Coefficient $C_1 = \frac{A_2}{A_1} \sqrt{2g}$
3. Set the height Initial Condition.
4. Run the Can problem for $1/4$ second.
5. Run the Square-Root computer until x^2 on Meter #2 equals h on Meter #1.
6. With Meter #1, measure both h and $-dh/dt$ and plot the results.
7. Repeat steps 4 through 6.

Figure 4.6
Single Can PAC Set Up



This incremental solution appears to be exactly what a digital computer does to solve problems. And so it is; however, we would have many more steps. By setting our Can Coefficient C_1 so that the can does not empty so fast, we really do nothing more than stretch out the time scale. Let us halve the input to the Can Integrator and halve our time scale so we can have twice as many increments.

Figure 4.7 shows that this results in a slightly longer time for the can to empty. It becomes evident that we have difficulty in getting accuracy down at small values of the square root.

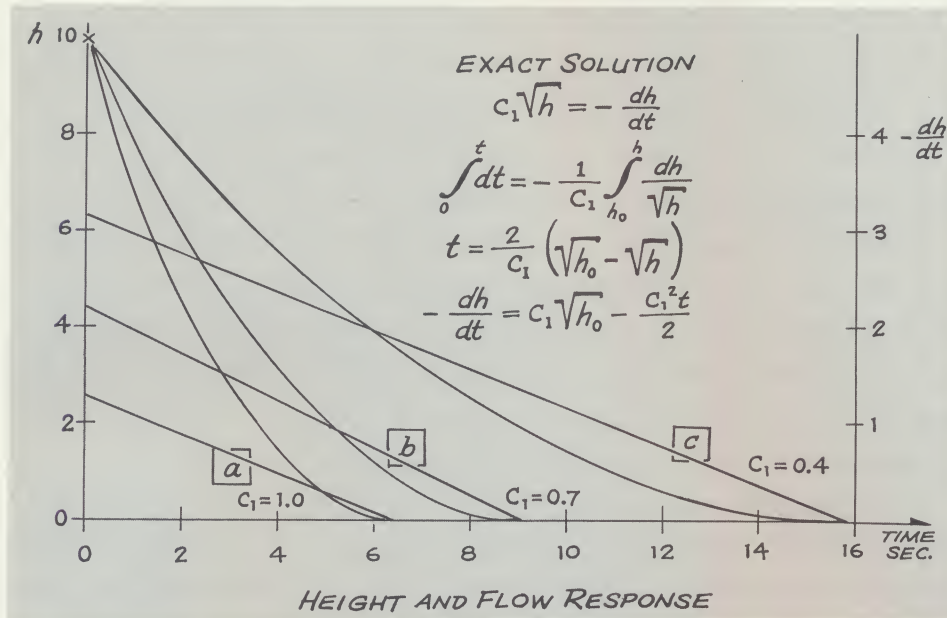


Figure 4.7
Solutions to Single Can Problem

INTERPRETATION OF THE SOLUTION

The plots of the solutions show that the flow variable changes linearly reaching zero at a definite time. We should compare this with the discharge of a capacitor. The capacitor current is proportional to its voltage, so it never completely discharges! A can flow is proportional to the square root of its head. Although the flow decreases with height, eventually the two variables h and dh/dt become less than 1, the flow continues to decrease but not as fast as the height, and the height eventually disappears. This is a subtle concept, like Zeno's paradox. It is a situation where linear thinking breaks down.

An Exact Solution

In considering the errors in our incremental solutions, it becomes evident that if only we could make the square rooter follow the height output exactly, we could have a perfect solution. Adjusting the can constant, it is possible to have a square root output which follows the height output. This must be possible, because we know that the flow input to the can is linear (if there is an incoming flow), the water height is the integral of a linear signal, and the square-root computer is made up of linear elements. Another way of looking at it is to notice in Figure 5 that $\sqrt{h}$ is followed by a Coefficient Multiplier and Integrator in both the square-root computer and in the height computer. We only have to adjust constants to have a perfect solution with a continuous square rooter.

The Two Can Solution

In facing the two can problem, we can consolidate the knowledge accumulated by the rather extended study in Lecture 4. Although the one can problem was relatively simple, it greatly clears the air for formulating the two can problem.

The major simplification that it provides is the fact that the flow from this can can be expressed as some initial flow plus a linearly decreasing factor. That is it starts out at some flow rate determined by how full the can is, and decreases linearly until it becomes zero. Mathematically then this flow can be expressed in terms of time and some constants dependent on initial height and can configuration.

This upper flow can be simulated by a single integrator, set at an initial condition representing initial flow, and a rate of integration to match the rate of flow decrease.

The flow from the second can is a function of height of water in that can and is expressed by the square root relationship formulated in Lecture #4. The change in height of the water in the second can will then be the sum of these flows:

$$\begin{array}{rcccl} & & \text{Negative} & & \\ \text{Decrease in height} & & \text{Flow from top} & & \text{Flow out bottom} \\ -dh'/dt & = & -C_2 + C_3 t & & + C_4 \sqrt{h'} \end{array}$$

In Figure 5.1 a computer interconnection is shown which represents this differential equation.

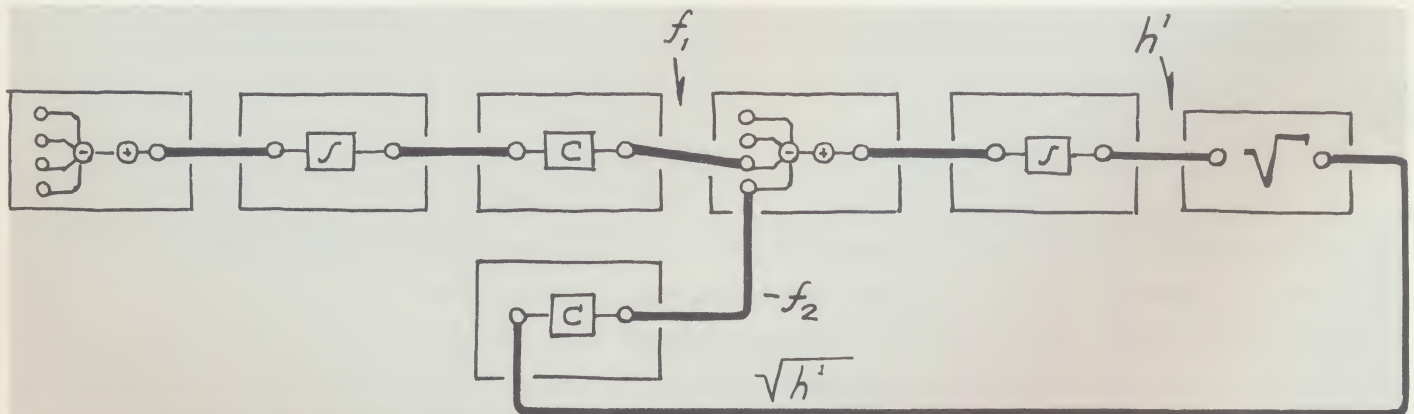


Figure 5.1
Computer Connections for the
Two Can Problem

The top can is what is called a signal source. It is in this case a flow source, analogous to a current source in electrical systems. The term source means that it is not influenced by the system it is supplying flow to, but puts out flow according to its own program. Sources usually simplify systems considerably because of this unilateral nature.

This system appears relatively simple, and the student should try to solve the differential equation by analytical methods. Certain solutions are possible, but are not general nor particularly useful in a real situation.

THE COMPUTER SOLUTION

A general computer solution is plotted out in Figure 5.2. Studying the curves will reveal many interesting facts which become self-evident, after seeing the solution.

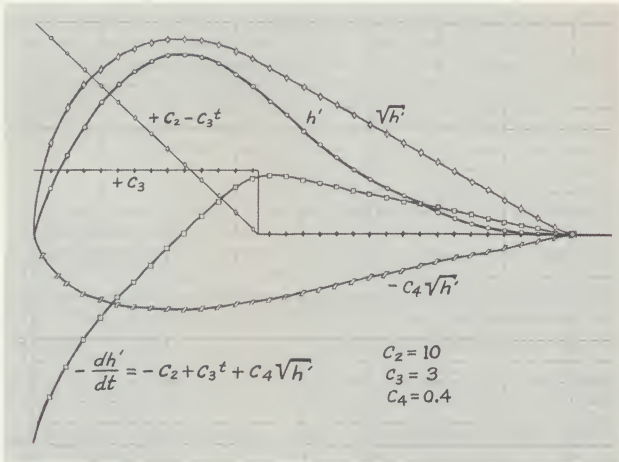


Figure 5.2
Some Solutions to the
Two Can Problem

The first step waveform C_3 represents the input to the top can Integrator unit. It causes this integrator to integrate negatively simulating exactly the decrease in flow of water from the top can into the bottom one. This step must be terminated when the top can reaches zero flow, because in real life the flow won't reverse.

The curve marked $C_2 - C_3t$ represents the actual flow schedule of the top can acting as a flow source. C_2 represents the initial flow determined by how high you fill the can.

h' is the height of the fluid in the bottom can at any time t and $C_4\sqrt{h'}$ is the flow resulting from that head out of the bottom of the can.

The curve $-dh'/dt$ representing the decrease in height of fluid in the bottom can is the algebraic summing accomplished by the Adder unit. It is seen to be negative, when the can is filling up, zero when the can reaches its maximum height, and positive as the can is emptying. It is expected that when the flow in from the top can is finally matched by the flow out from the bottom can, the height will be a maximum, which is the solution we are looking for.

After the flow from the top can has stopped the flow from the bottom can which is proportional to h' will become linearly decreasing exactly like the one can problem. Therefore this curve is not continuous in all its derivatives. It has suddenly no second derivative at

this point, although it clearly had one earlier. It cannot be expressed as a analytical function, so we would not expect to find a solution to the problem by analytical means.

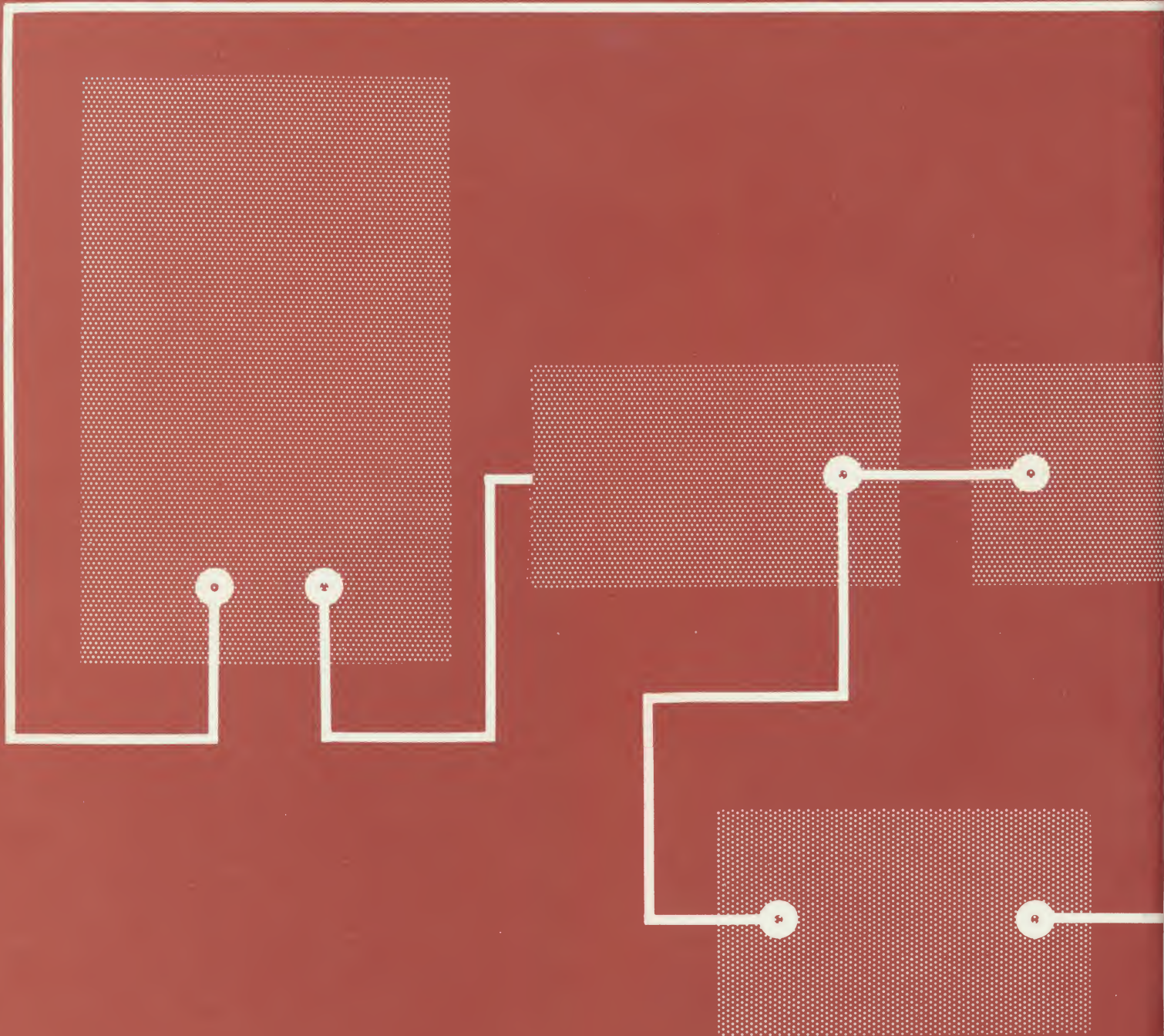
As a further investigation, the student may discover what the relationship between the initial height in the top can is to the maximum height in the bottom can, so as to be able with only one trial to determine what height in the top can will almost but not quite overflow the bottom can.

Conclusion

We have described a method of reticulation and analysis suggested by Professor Henry Paynter. It has many more implications than can possibly be appreciated from this brief paper. Beyond providing a firm and consistent method of exhibiting the structure of a complex system, it permits the engineer to program such a system for analog or digital computer analysis. It lends itself to a notation by which systems structures can be communicated to computers. It yields easily to the methods of matrix solution used in digital computers as well as direct modeling which we have suggested with PAC.

The subject matter extends far beyond the elementary topics covered in these lectures. Perhaps the most important subject is the multiport concept where more than one pair of effort and flow variables feed into an element, such as a valve. Such elements are modeled by multipliers and other analog elements not generally available in the Standard PAC units.

It is to be hoped that the inevitable limitations in this presentation as well as those in the PAC equipment will encourage the student to venture further into the realm of computers and systems engineering. If he does so he should have a firmer foundation and greater enthusiasm.



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